How to Design High Order Filters with Stopband Notches Using the LTC1562 Quad Operational Filter (Part 1)

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This is the first in a series of articles describing applications of the LTC1562 connected as a lowpass, highpass or bandpass filter with added stopband notches to increase selectivity. Part 1 covers lowpass filters.

Lowpass filters with stopband notches are useful in applications seeking steep attenuation in the vicinity of the cutoff frequency. When compared to classical all-pole realizations (such as Butterworth or Chebyshev) they are more "efficient"; that is, they meet a given attenuation requirement with the least number of poles.

Lowpass filters with stopband notches (broadly referred to as "elliptics" or "Cauers") can be designed with the aid of some literature or with commercially available software. The new FilterCAD™ for Windows® program, supplied free of charge by Linear Technology Corporation, is an excellent example.

For instance, a 100kHz lowpass filter with a 0.1dB passband ripple and 40dB attenuation at 200kHz can be realized with a 6th order Chebyshev or a 4th order textbook elliptic. Curves A and B of Figure 1 illustrate the respective amplitude responses.

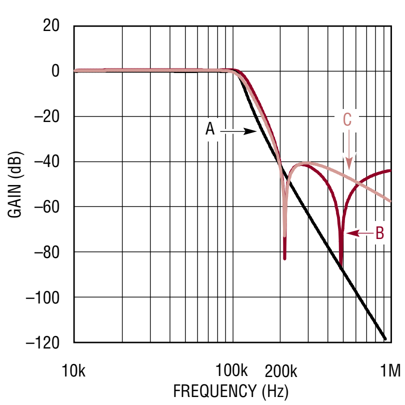

Figure 1. Amplitude responses of 6th order Chebychev (A), 4th order textbook elliptic (B) and modified 4th order elliptic (C) lowpass filters

When considering the practical implementation of the filters of Figure 1 (curves A and B), in the author's experience, it is easier to implement the higher order all-pole filter (curve A), rather than the 4th order version with the stopband notches. The realization of deep stopband notches may result in hardware complexity. This is especially true if a discrete R-C active implementation is chosen and if a single 5V supply and a wide input dynamic range are required.

Nevertheless, curve C of Figure 1 is of particular interest because of its rather simple hardware implementation. Curve C is derived from the classical elliptic response, curve B, where the high frequency notch is "pushed" to infinity and the highest Q pole pair is readjusted to maintain passband flattness. The penalty is the slight gain roll-off at the cutoff frequency, which, for many applications, is acceptable. For sake of simplicity the amplitude response of the filter of Figure 1, curve C, is called a "p-e" (pseudo-elliptic) response.

Figure 2 illustrates the group delay responses of the three filters of Figure 1, with the same curve letter designations. The group delay of curve C is the flattest.

Hardware Implementation

High order filter realizations were a subject of passionate interest in the 1960s and '70s. One very popular method, which stems from the simplicity of its hardware implementation, consists of decomposing a high order filter polynomial into cascaded second and first order polynomials. Each polynomial is then implemented with commercially available active and passive components. The major drawback of the "cascaded" method is the relatively high Q of at least one of the 2nd order sections and the resulting requirement for precision components for its realization.

Using the "cascading" principle outlined above, the "p-e" response of curve c, Figure 1, can be treated as a self-contained 4th order block and two (or more) of these blocks can be cascaded to form an 8th order (or higher) lowpass filter with two (or more) stop-band notches. This interesting novelty is driven by the simplicity of its hardware realization; it requires, however, the transformation of an 8th order classical elliptic lowpass response into two cascaded 4th order "p-e" responses.

Figure 3 shows a compact hardware implementation of the 4th order "p-e" filter using one half of the LTC1562 quad Operational Filter IC, which was introduced in the February 1998 issue of *Linear Technology* magazine.[1] Two 2nd order sections form the 4th order filter function. A phase-shifting external capacitor, C_{IN1}, and a feedforward path through resistor R_{FF2}, create the desired notch.

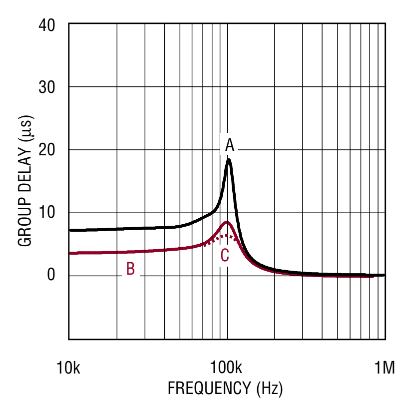

Figure 2. Group-delay responses corresponding to the amplitude responses of Figure 1.

To make the circuit technique of Figure 3 intuitively obvious, consider the following:

A signal of a given frequency can be notched if it is phase shifted by 180 degrees and then summed with itself. If the summation is governed by equal gains, a complete signal cancellation occurs and the notch depth, at least in theory, is infinite.

A phase shift of 180 degrees at a single frequency, f_O, is easily provided by a second order inverting bandpass filter; hence, in Figure 3, if C_{IN1} equals zero, a notch is formed as the bandpass output (pin 2) is summed with the input via (R_{IN2}, R_{FF2}). Moreover, if the summation has equal gains (1), the notch should, in theory, have infinite depth.

$$R_{FF2} = R_{IN2} \bullet (R_{IN1}/R_{Q1}) \qquad (1)$$

In Figure 3, an external capacitor, C_{IN1}, is added to provide additional phase lead, so that the frequency of the notch is higher than the center frequency, f_{O1}, of the second order section used to create it.

The notch frequency, f_{n1}, is directly proportional to the center frequency, f_{O1}, and indirectly proportional to the time constant ($R_{IN1} \bullet C_{IN1}$) divided by the ($R_{Q1} \bullet C$) (C is an internal capacitor of 159pF); therefore:

$$f_{n1} = f_{01}\sqrt{1 - \frac{\dfrac{1}{R_{IN1} \bullet C_{IN1}}}{R_{Q1} \bullet C}} \qquad (2)$$

A step-by-step algorithm for building compact "p-e" lowpass filters with the new LTC1562 quad Operational Filter building block is outlined below:

Start with a set of two (lowpass) pole pairs and one finite stopband notch. Arrange the pole pairs in ascending order of Q values.

Example 1:

f_{O1} = 113.76kHz, Q1 = 2.28, f_{n1} = 224.7kHz, f_{O2} = 85.8kHz, Q2 = 0.64, $f_{n2} = \infty$;

1. Calculate the frequency-setting resistor, R21:

   $R21 = (100kHz/f_{01})^2 \bullet 10k\Omega$
   (choose the closest 1% value)
   R21 = 7.68k (1%)

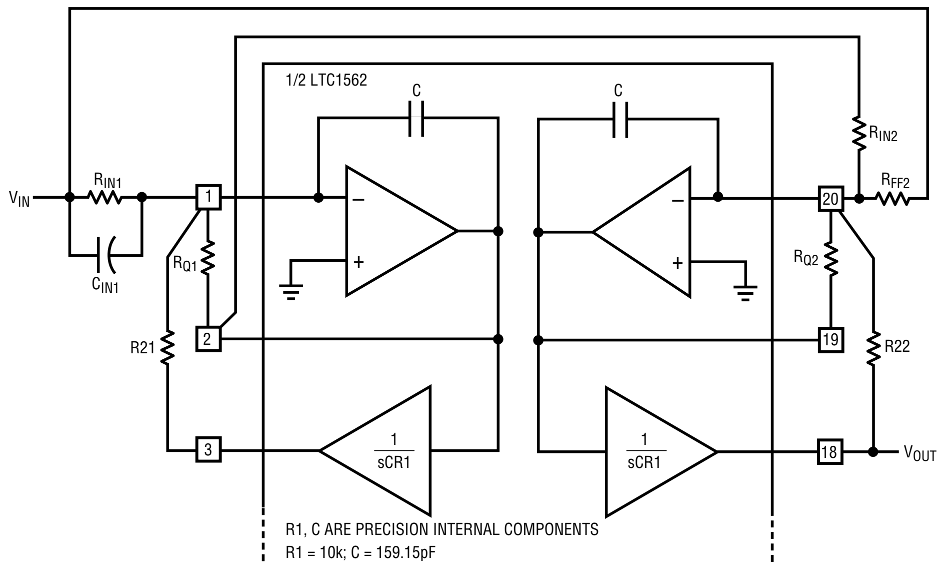

Figure 3. Hardware implementation of a 4th order pseudo-elliptic filter using one-half of an LTC1562 quad Operational Filter IC

2. Calculate the Q-setting resistor, R_{Q1}:

   $R_{Q1} = Q1 \bullet \sqrt{(R21 \bullet 10k)}$
   (choose the closest 1% value)
   R_{Q1} = 20k (1%)

Note: The calculations for R21 and R_{Q1} are from the LTC1562 Data Sheet; they are applicable to any 2nd order section using the LTC1562 proprietary architecture.

3. Calculate the input resistor, R_{IN1}, from the following expression:

$$R_{IN1} = Q1 \bullet \sqrt{\left[R_{Q1} \bullet \left(1 - \left(\frac{f_{01}}{f_{n1}}\right)^2\right)\right]^2 + (R21 \bullet 10k\Omega)}$$

R_{IN1} = 39.36k

Make sure that R_{IN1} > R21; If not, make R_{IN1} = R21 and proceed to Step 4a.

Note: $R21/R_{IN1}$ is the DC gain from the input to the lowpass output of the first building block, pin 3. The expression for R_{IN1} ensures optimum dynamic behavior of all nodes of the LTC1562.

4a. Use the value of R_{IN1}, calculated above, and calculate the value of the input capacitor C_{IN1} from the notch equation (2).

   $C_{IN1} = 159.15pF \bullet \{(R_{Q1}/R_{IN1}) \bullet [(1 - (f_{01}/f_{n1})^2]\}$
   C_{IN1} = 60.14pF

Use a commercially available NPO-type 0402 surface mount capacitor with the value nearest the ideal value of C_{IN1} calculated above. For instance, if $C_{IN1\ (ideal)}$ is 60.14pF, choose an off-the-shelf 56pF standard value.

4b. Recalculate the value of R_{IN1} after a C_{IN1} of 56pF is chosen.

   $R_{IN1} = (C_{IN1(ideal)} \bullet R_{IN1(ideal)})/C_{IN(NPO,\ 0402)}$
   (choose the closest 1% value)
   R_{IN1} = 42.2k (1%)

5. Calculate the frequency- and Q-setting resistors R22, R_{Q2}, as done in steps 1 and 2, above. Choose the closest 1% standard resistor values.

   R22 = 13.3k (1%); R_{Q2} = 7.32k (1%)

6. Calculate the feedforward resistor, R_{FF2}:

   R_{FF2} = R22/(DC gain),
   R_{FF2} = 13.3k
   DC gain = V_{OUT}/V_{IN} (DC) = 1

7. Calculate the input resistor R_{IN2}, to satisfy the gain condition for the notch (1).

   $R_{IN2} = R_{FF2} \bullet (R_{Q1}/R_{IN1})$
   R_{IN2} = 6.34k (1%)

Make the practical value of R_{IN2} as close as possible to the value calculated above; otherwise, the stopband notch depth will be affected.

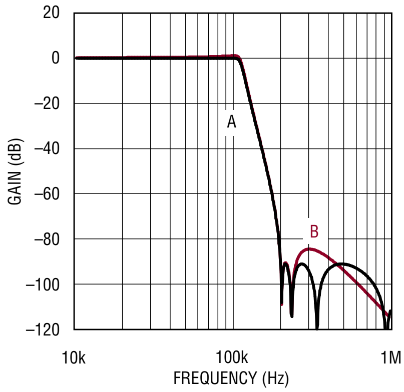

Figure 4. Amplitude responses of 8th order 100kHz elliptic lowpass filters comprising four cascaded 2nd order biquad sections (A) and the filter after the two highest notch frequencies are raised to infinity

An Example Using FilterCAD

The following is a comprehensive example of how to synthesize and realize a complex lowpass filter using two "p-e" 4th order sections in cascade. FilterCAD for Windows will be used to synthesize the filter.

A classical 8th order, 100kHz lowpass elliptic filter with theoretical passband ripple, A_{MAX}, of 0.005dB, and a minimum stopband attenuation, A_{MIN}, of 85dB at twice cutoff, can be synthesized by cascading four biquadratic 2nd order sections, as shown in Table 1. Each biquadratic section comprises a complex pole pair of center frequency f_O, and an imaginary zero pair of notch frequency f_n. The amplitude response is shown in Figure 4, curve A. The filter above is easily transformed into two cascadable 4th order "p-e" sections by performing the following steps.

1. Set the two highest notch frequencies to infinity and expect a decrease in stopband attenuation as well as gain peaking in the vicinity of the cutoff frequency (Figure 4, curve B).
2. Use the interactive capability of FilterCAD to increase the frequency of the right hand notch (Figure 5 curve C), until the stopband ripple has equal peaks.
3. Use the interactive capability of FilterCAD to flatten the passband by lowering the Qs. Start with the highest Q, then proceed with the second highest, then the third.

Table 2 illustrates the parameters of the transformed filter. Compared to Table 1, two notch frequencies are set to infinity, one notch frequency has been increased and the three highest Qs have been reduced. Figure 5, curve C, illustrates the amplitude response of the transformed filter. The original filter shown in Figure 4 is also shown in Figure 5, curve A, for comparison. The main difference between curves A and C is the theoretical stopband attenuation. Curve C, with its lower Q, will also exhibit improved transient behavior.

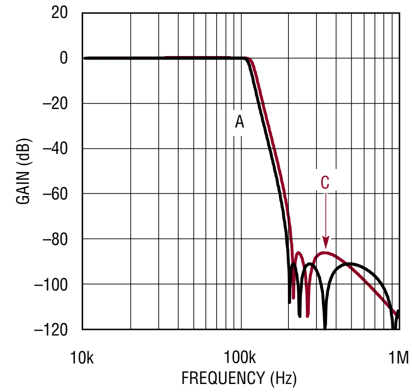

Figure 5. Comparison of original 8th order 100kHz elliptic filter (A) and filter transformed by FilterCAD into two cascaded 4th order "p-e" sections (C)

A Practical Case

The high Qs of the previous synthesized filters ensure, at least in theory, passband flatness all the way up to the cutoff frequency. In practice, errors occur in the vicinity of the filter cutoff. They are most often manifested as gain peaking and they are caused by the tolerances of the passive components and the finite bandwidth of the active circuitry. The gain peaking at the filter cutoff can be addressed by predistorting the high Q section, that is, by intentionally lowering the Q so that the theoretical response will show some gain rolloff at the cutoff frequency.

The synthesized filter of Table 2 can be efficiently realized by two cascaded "p-e" 4th order sections, as illustrated in the block diagram, Figure 6. Note the arrangement of the pole-zero pairs of Figure 6 and compare it with Table 2. In Table 2, the sections appear in order of increasing f_O and Q. In Figure 6, within each 4th order "p-e" filter, the 2nd order section with the highest Q is placed first; the 4th order "p-e" filter containing the highest Q is cascaded last. The notches (f_{n1} and f_{n2}) are so arranged that the highest frequency notch is formed from the pole pair whose center frequency (f_O) is closest to the filter cutoff frequency. For example, the 250kHz notch is placed with the 99.99kHz pole pair. This nonobvious arrangement allows for a stopband attenuation approaching the theoretical values. The highest Q of 5.87 is

Table 1. Parameters of 8th order, 100kHz lowpass elliptic filter synthesized by cascading four biquadratic 2nd order sections

f_0	Q	f_n
61.8049e3	0.5471	957.9224e3
81.2817e3	0.9230	343.0259e3
99.9948.e3	1.9047	235.4796e3
109.8890e3	6.4428	203.3896e3

Table 2. Parameters of Table 1's filter transformed into two cascadable, 4th order "p-e" sections

f_0	Q	f_n
61.8000e3	0.5471	∞
81.2800e3	0.9046	∞
99.9900.e3	1.7555	250.6400e3
109.8800e3	5.874	203.3900e3

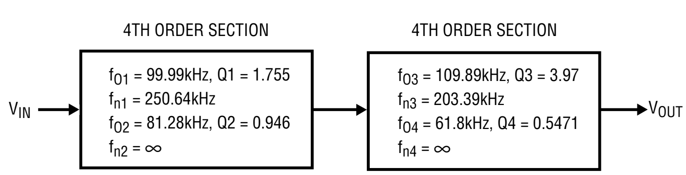

Figure 6. Realization of Table 2's filter with two cascaded 4th order "p-e" sections

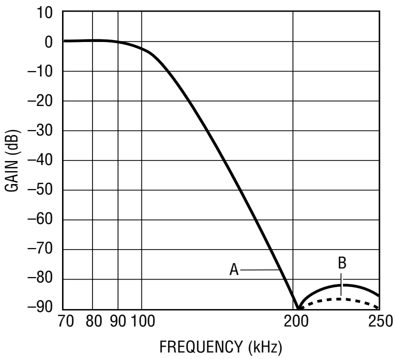

Figure 7. Amplitude response of Figure 7's filter before (A) and after (B) R_{Q1} was lowered to 16.2k to better define the notch.

reduced to 3.97 for reasons mentioned above and for improving the transient response of the circuit. See Figure 7 for the amplitude response; note the slight rolloff at the cutoff frequency. Figure 8 shows the complete hardware realization using all four sections of an LTC1562 continuous-time quad Operational Filter IC. The algorithm outlined above was followed to calculate the values of the external passive components. The circuit occupies as much real estate as a U.S. dime. This is quite significant considering the cumbersome alternative of a fully discrete realization with op amps, Rs and Cs.

Experimental Results

Figure 7, curve A, shows the amplitude response of the filter hardware illustrated in Figure 8. No attempt was made to adjust any component. Both notches are fully resolved, but due to the tolerances of the components and the finite bandwidth of the active circuitry, the stopband attenuation, although impressive, is 2dB above the theoretical value. Subsequently, the value of R_{Q1} was lowered to 16.2k (curve B) to better define the first notch. The filter reaches attenuation levels beyond 85dB all the way up to 0.5MHz input frequencies. The measured attenuation at 1MHz was still better than 78dB. The dynamic range of the circuit is quite impressive: the measured wideband noise was $40\mu V_{RMS}$ and the THD for $1V_{RMS}$ and 50kHz input signal was better than –80dB.

[1] Hauser, Max. "Universal Continuous-Time Filter Challenges Discrete Designs." *Linear Technology* VIII:1 (February 1998), p.1.

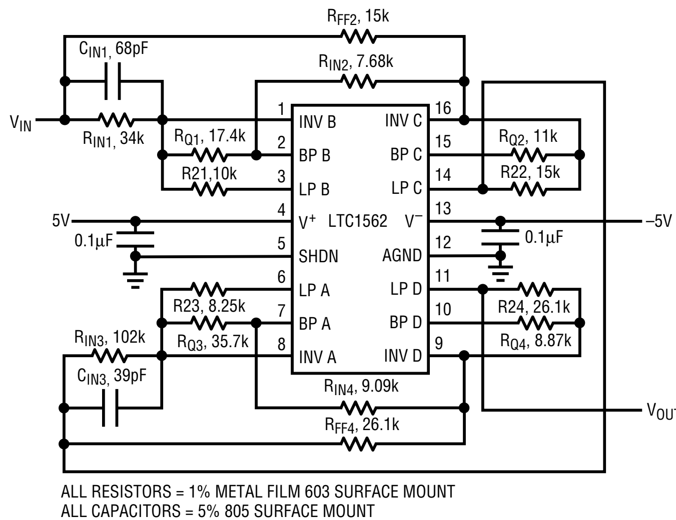

Figure 8. Hardware realization of the filter in Figure 6, using all four sections of an LTC1562.