mobile communications series

# SOFTWARE-DEFINED RADIO for ENGINEERS

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*Dedication*

To my wife Lauren
—Travis Collins

To my wonderful children, Matthew, Lauren, and Isaac, and my patient wife, Michelle—sorry I have been hiding in the basement working on this book. To all my fantastic colleagues at Analog Devices: Dave, Michael, Lars-Peter, Andrei, Mihai, Travis, Wyatt and many more, without whom Pluto SDR and IIO would not exist.
—Robin Getz

To my lovely son Aidi, my husband Di, and my parents Lingzhen and Xuexun
—Di Pu

To my wife Jen
—Alexander Wyglinski

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APPENDIX C

# Equalizer Derivations

## C.1 Linear Equalizers

Suppose we assume a transceiver model where the information source produces amplitude values $I_n$ applied to an infinite impulse train; namely,

$$s(t) = \sum_{n=-\infty}^{\infty} I_n \delta(t - nT), \tag{C.1}$$

where $\delta(t)$ is the Dirac delta function. Applying a transmit pulse shaping filter $h_T(t)$ to the information signal $s(t)$, we obtain the transmitter's output signal:

$$v(t) = \sum_{n=-\infty}^{\infty} I_n h_T(t - nT), \tag{C.2}$$

which is then sent through a propagation channel that is characterized by a channel filter $h_C(t)$ and an additive white Gaussian noise signal $z(t)$. The output of the channel filter yields a signal:

$$p(t) = \sum_{n=-\infty}^{\infty} I_n h(t - nT), \tag{C.3}$$

where $h(t) = h_T(t) * h_C(t)$ is the channel impulse response. The signal intercepted by the receiver is expressed as

$$r(t) = p(t) + z(t) = \sum_{n=-\infty}^{\infty} I_n h(t - nT) + z(t). \tag{C.4}$$

At the receiver, we would like to find the expression for the mean squared error (MSE) and minimize it. To achieve this objective, we choose the receive filter $h_R(t)$ to be matched to $h(t)$, yielding the following optimal result:

$$h_R(t) = h^*(-t), \tag{C.5}$$

which results in the output of the receiver filter being equal to

$$y(t) = p(t) * h^*(-t) + z(t) * h^*(-t) = \sum_{n=-\infty}^{\infty} I_n g(t - nT) + z(t) * h^*(-t), \tag{C.6}$$

where $g(t) = h(t) * h_R(t)$, and

$$x(t) = \phi_h(t) = h(t) * h^*(-t) = \int_{-\infty}^{\infty} h(t+\tau)h^*(\tau)d\tau. \tag{C.7}$$

Supposed that $v(t) = z(t) * h^*(-t)$, then we get the following expression:

$$y(t) = \sum_{n=-\infty}^{\infty} I_n x(t - nT) + v(t). \tag{C.8}$$

Now, let us sample the output of the receive filter such that

$$y_k = y(kT) = \sum_{n=-\infty}^{\infty} I_n x((k-n)T) + v(kT), \tag{C.9}$$

$$= \sum_{n=-\infty}^{\infty} I_n x_{k-n} + v_k. \tag{C.10}$$

Note that $x_k$ is referred to as the channel autocorrelation. Furthermore, the $v_k$ samples are not white due to the filtering by $h^*(t)$, which means that $E\{v_k v_{k+n}\} \neq \delta(n)$. In these circumstances, we have a couple of options to deal with this colored noise. One approach involves finding $E\{v_k v_l^*\}$ for the noise sequence $v_k$. In this approach, we know that $v_k(t)$ is Gaussian since $z(t)$ is Gaussian. Furthermore, we know that $z(t)$ is white; that is, $S_z(f) = N_0$. Thus, if $E\{z^{ast}(s)z(t)\} = N_0\delta(t-s)$, we can then solve $E\{v_k v_l^*\}$ as follows:

$$E\{v_k v_l^*\} = \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dt h(t - lT)h^*(s - kT)E\{z^*(s)z(t)\}, \tag{C.11}$$

$$= N_0 \int_{-\infty}^{\infty} dt h(t - lT)h^*(t - kT), \tag{C.12}$$

$$= N_0 \int_{-\infty}^{\infty} dt h(t + |k - l|T)h^*(t). \tag{C.13}$$

Since we have:

$$x_k = \int_{-\infty}^{\infty} h^*(t)h(t + kT)dt, \tag{C.14}$$

which then gives us the expression for $E\{v_k v_l^*\}$ as:

$$E\{v_k v_l^*\} = N_0 x_{k-l}, \tag{C.15}$$

that can then be written as the autocorrelation function and the power spectral density of $v_k$; namely,

$$R_v(k) = E\{v_n v_{k+n}^*\} = N_0 x_k, \tag{C.16}$$

$$S_v(z) = \mathcal{Z}\left\{R_v(k)\right\} = N_0 X(z). \tag{C.17}$$

The second approach for dealing with colored noise is to implement something referred to as a *whitening filter*. In this case, we try to reverse the effects of the receiver filter on the noise signal $z(t)$. In other words, since we have the power spectral density of $v_k$ equal to $S_v(z) = N_0 X(z)$, we ultimately would like to have the whitened noise power spectral density only equal to $N_0$. To achieve this, we assume that the z-transform of $x_k$, $X(z)$, can be represented by the following:

$$X(z) = F(z)F^*(1/z^*). \tag{C.18}$$

Thus, in we have a whitening filter whose transfer function is equal to $1/F^*(1/z^*)$, the resulting power spectral density at the output of this whitening filter should be equal to:

$$S_n(z) = \frac{1}{F(z)F^*(1/z^*)} S_v(z) = \frac{N_0 X(z)}{X(z)} = N_0, \tag{C.19}$$

which yields an output noise signal that is white.

## C.2 Zero-Forcing Equalizers

Suppose we assume a discrete time model for the receiver that is equal to the following:

$$w_k = \sum_{n=-\infty}^{\infty} I_n f_{k-n} + n_k, \tag{C.20}$$

where $w_k$ is the output signal of the whitening filter, $f_k$ is the impulse response of the whitening filter, and $n_k$ is the whitened noise signal with power spectral density equal to $N_0$.

In the *zero-forcing equalizer* (ZFE), we choose a transfer function $C(z)$ such that each ISI-compensated term $q_k = \delta_k$, which implies that there is no ISI present and the resulting equalized outputs $I_k + n'_k$ are subsequently quantized, yielding a probability of error equal to $P_e = Q(1/\sigma')$ for $I_k = \pm 1$. Although $P_e$ is often used as a performance metric for a communication system, in the case of equalizer design we are going to use the minimum mean squared error (MMSE) as our metric; that is, $E\{|I_k - \hat{I}_k|^2\}$.

Suppose we take the z-transform of (C.20), thus obtaining:

$$W(z) = I(z)F(z) + N(z) \tag{C.21}$$

and once this has been filtered by the ZFE equalizer we get the output equal to

$$W(z)C(z) = I(z)F(z)C(z) + N(z)C(z), \tag{C.22}$$

with $Q(z) = F(z)C(z)$. The objective of our ZFE is to generate an output equal to

$$\mathcal{Z}^{-1}\{W(z)C(z)\} = I_k * q_k + n'_k, \tag{C.23}$$

where $q_k = \delta_k$; that is, no ISI present in the signal. Consequently, the ISI term in this expression is $F(z)C(z)$ and thus we want it to be equal to $Q(z) = F(z)C(z) = 1$ for $q_k = \delta_k$. Rearranging the terms, our ZFE should be equal to $C(z) = 1/F(z)$.

## C.3 Decision Feedback Equalizers

Although ZFE filters are conceptually straightforward, they almost always possess an infinite number of taps, thus making them impossible to implement in real world applications. Furthermore, when the signal resulting at the output of the whitening filter is equalized using the ZFE, any noise contributions contained within the equalized signal will no longer be white. As a result, there exists other equalizer designs that can be more readily implemented in hardware. One of these is the *decision feedback equalizer* (DFE), which consists of two filters: a feedforward filter $a(z)$ and a feedback filter $b(z) - 1$.

The *feedforward filter* $a(z)$ is anti-causal and has the following form:

$$a(z) = a_{-1}z + a_{-2}z^2 + a_{-3}z^3 + a_{-4}z^4 + \ldots, \tag{C.24}$$

while the *feedback filter* $b(z)$ is a causal filter possessing the form:

$$b(z) - 1 = b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + \ldots. \tag{C.25}$$

Consequently, we can design $a(z)$ and $b(z)$ such that

$$a(z) = CF^*(1/z^*) \tag{C.26}$$

$$b(z) = F(z) \tag{C.27}$$

$$F(z)F^*(1/z^*) = X(z) + N_0. \tag{C.28}$$